Game-Theoretic Control under Different Classes of Restrictions on Pursuer and Evader

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Usually it is considered that the pursuer and the evader are restricted by constraints of the same kind when dealing with the problem of game-theoretic control synthesis. For example, both are subject to geometric or integral constraints. This, however, is not always the case in real applications. This report is devoted to the problem of control strategy synthesis for differential games where the pursuer and the evader are subject to geometric and integral constraints respectively (and vice versa).

Solution is based on combination of dynamic programming techniques and convex analysis. Additional variable is introduced to track how evader uses its reserve of the integral constraint. Though solvability domain is not convex even in simplest cases, its sections by constant reserve plane are always convex, which justifies the use of convex analysis tools. With mentioned reserve variable the integral constraint of evader becomes a state space constraint. An analogue of Pontryagin's alternated integral is developed, which is proved to be the solution of an evolution equation. Solvability domain satisfies the optimality principle in form of the semigroup property. Pursuer's control strategy is built using extremal aiming rule using the solvability domain (or the lower alternated integral, in case when they do not coincide). Hamilton–Jacobi–Bellman–Isaacs equation is obtained for the value function. It is shown that the value function is not greater than the distance to the solvability domain section; in the case of one dimension this becomes equality so that control strategies obtained through the extremal aiming rule and through the value function coincide.